

MAGNITOELASTIC INTERACTION AND LONG-RANG MAGNETIC ORDERING IN TWO-DIMENSIONAL FERROMAGNETICS

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Abstract

The influence of magnitoelastic (ME) interaction on the stabilization of long-range magnetic order (LMO) in the two-dimensional easy-plane ferromagnetic is investigated in this work. The account of ME exchange results in the root dispersion law of magnons and appearance of ME gap in the spectra of elementary excitations. Such a behavior of the spectra testifies to the stabilization of LMO and finite Curie's temperature.

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I. INTRODUCTION

It is well known [1, 2], that the existence of the long-range magnetic order (LMO) in 2D ferromagnetic is impossible. Formally it means, that in a Heisenberg ferromagnetic the integral

$$\langle \Delta M \rangle \propto \int_0^\omega N(\omega) k dk,$$

where $N(\omega) = [\exp(\omega/T) - 1]^{-1}$, $\omega \propto k^2$ is the law of dispersion of spin waves, which determines fluctuations of the magnetic moment, diverges on the lower limit.

Berezinskii, Kosterlitz and Thouless [3, 4] have shown, that in 2D easy-plane magnetics at finite temperatures there can exist only a quasilong-range order, that structure is determined rather by magnetization vortices, then by spin waves.

However in [5] it was shown, that the account of magnetodipole interaction in 2D ferromagnetics results in the root dispersion law of magnons $\omega \propto \sqrt{k}$. It causes the convergence of the integral $\langle \Delta M \rangle$ and testifies to stabilization of LMO at $T < T_c$, where T_c is the temperature of the phase transition. Proceeding from this, it is possible to assume, that other relativistic types of interactions can stabilize LMO. For example, in paper [6], it is shown, that in an easy-plane 2D antiferromagnetic LMO is stabilized by magnitoelectric (ME) interaction. However in this work there is used the Holstein-Primakoff representation for spin operators, i.e. quasiclassical representation, which is obviously inadequate to the microscopcity of the system in study.

Therefore it is of interest to investigate a question of stabilization of LMO in 2D easy-plane ferromagnetic with the exact account of one-ion anisotropy (OA) and ME interaction. Such an account can be made with the use of the Hubbard's operators procedure [7, 8]. This technique allows us to obtain the dispersion equation of hybridized magnitoelectric elementary excitations (bound ME waves), valid for arbitrary values of a magnetic ion spin ($S \geq 1$), temperatures (up to temperature of phase transition), and anisotropy constant.

II. MODEL SYSTEM

As a model system, we shall consider an easy-plane 2D ferromagnetic (XOZ -basic plane), with the Hamiltonian:

$$H = -\frac{1}{2} \sum_{n,n'} I(n - n') \vec{S}_n \vec{S}_{n'} + \frac{\beta}{2} \sum_n (S_n^y)^2 + \lambda \sum_n \{ (S_n^x)^2 u_{xx} + (S_n^z)^2 u_{zz} + (S_n^x S_n^z + S_n^z S_n^x) u_{xz} \} + \int dv \frac{E}{2(1 - \sigma^2)} [u_{xx}^2 + u_{zz}^2 + 2\sigma u_{xx} u_{zz} + 2(1 - \sigma) u_{xz}^2], \quad (1)$$

where S_n^i is the spin operator in site n , $\beta > 0$ is the OA constant, λ is the constant of ME interaction, u_{ij} is a symmetric part of tensor of deformations, E is the modulus of elasticity, σ is the Poisson coefficient. To simplify calculations we shall assume, that a magnetic ion spin $S = 1$.

In the Hamiltonian (1) the first two terms describe the magnetic subsystem, the third is the two-dimensional ME interaction, and the fourth is the elastic subsystem.

As we study the possibility of stabilization of LMO due to the presence of ME exchange let us suppose that ME interaction creates the nonzero magnetic moment, which for definiteness, we consider parallel to OZ axis.

After separation of the mean field in the exchange part of (1), we obtain the one-ion Hamiltonian:

$$H_0(n) = -I_z S_n^z + \frac{\beta}{2} (S_n^y)^2 + \lambda [u_{xx} (S_n^x)^2 + u_{zz} (S_n^z)^2 + u_{xz} (S_n^x S_n^z + S_n^z S_n^x)]. \quad (2)$$

Solving the one-ion problem with the Hamiltonian (2) we find the energy levels of a magnetic ion:

$$E_1 = \frac{\beta}{4} + \frac{\lambda}{2} (u_{xx}^{(0)} + 2u_{zz}^{(0)}) - \chi, \quad E_0 = \frac{\beta}{2} + \lambda u_{xx}^{(0)}, \quad E_{-1} = \frac{\beta}{4} + \frac{\lambda}{2} (u_{xx}^{(0)} + 2u_{zz}^{(0)}) + \chi, \quad (3)$$

$$\chi^2 = I_z^2 + \left(\frac{\lambda}{2} u_{xx}^{(0)} - \frac{\beta}{4}\right)^2, \quad I_z = I(0) < S^z >, \quad \text{and eigenfunctions of } H_0(n):$$

$$\Psi(1) = \cos \delta |1\rangle + \sin \delta |-1\rangle, \quad \Psi(0) = |0\rangle, \quad \Psi(-1) = -\sin \delta |1\rangle + \cos \delta |-1\rangle, \quad (4)$$

where

$$\cos \delta = \left(\frac{\lambda u_{xx}^{(0)}}{2} - \frac{\beta}{4} \right) \left[(\chi - I_z)^2 + \left(\frac{\lambda u_{xx}^{(0)}}{2} - \frac{\beta}{4} \right)^2 \right]^{-1/2},$$

and $|i\rangle$ is the eigenvector of the operator S^z , $u_{ij}^{(0)}$ are spontaneous deformations determined from the condition of the free energy density minimum: $F = F_e - T \ln Z$, where F_e is the density of an elastic energy, $Z = \sum_M \exp(-E_M/T)$ is the partition function. In our case the spontaneous deformations have the form:

$$u_{xx}^{(0)} = -\frac{\lambda}{E} \cdot \frac{1-2\sigma}{2}, \quad u_{zz}^{(0)} = -\frac{\lambda}{E} \cdot \frac{2-\sigma}{2}, \quad u_{xz}^{(0)} = 0.$$

Note, that the calculations are carried out in a low temperature limit (the temperature is much less than the temperature of the phase transition). In this case it is possible to account only the lowest energy level, which as it follows from (3), is E_1 .

In the basis of eigenfunctions of the operator $H_0(n)$ we shall build the Hubbard's operators $X_n^{MM'} \equiv |\Psi_n(M')\rangle \langle \Psi_n(M)|$, which describe transitions of a magnetic ion from a state M' to a state M . These operators are connected with the spin operators by the relations:

$$S_n^+ = \sqrt{2} \cos \delta \cdot (X_n^{10} + X_n^{0-1}) + \sqrt{2} \sin \delta \cdot (X_n^{01} - X_n^{-10}),$$

$$S_n^- = (S_n^+)^+, \quad (5)$$

$$S_n^z = \cos 2\delta \cdot (H_n^1 - H_n^{-1}) - \sin 2\delta \cdot (H_n^{1-1} - H_n^{-11}),$$

where $S_n^\pm = S_n^x \pm i S_n^y$, $H_n^M \equiv X_n^{MM}$ are diagonal Hubbard's operators.

In terms of Hubbard's operators the one-ion Hamiltonian can be represented in the form

$$H_0(n) = (\vec{E} \vec{H}_n) + \lambda u_{xz} [d_1 (X_n^{01} + X_n^{10}) + d_2 (X_n^{0-1} + X_n^{-10})],$$

where we denote:

$$d_1 = \frac{1}{\sqrt{2}}(\cos\delta - \sin\delta), \quad d_2 = -\frac{1}{\sqrt{2}}(\cos\delta + \sin\delta).$$

Further, we represent components of tensor of deformations in the form $u_{ij} = u_{ij}^{(0)} + u_{ij}^{(1)}$, where $u_{ij}^{(1)}$ is the dynamic part of tensor of deformations describing oscillations of a lattice. Applying to $u_{ij}^{(1)}$ the method of harmonic quantization [9], from the Hamiltonian (2) we shall obtain the Hamiltonian, describing processes of transformations of magnons to phonons and vice versa:

$$H_{tr} = \sum_n [\sum_M P_M H_n^M + \sum_\alpha P_\alpha X_n^\alpha], \quad (6)$$

$$P_{M(\alpha)} = \frac{1}{\sqrt{N}} \sum_{q,\lambda} (b_{q,\lambda} + b_{-q,\lambda}^\dagger) T_n^{M(\alpha)}(q, \lambda).$$

Here $b_{q,\lambda}(b_{-q,\lambda}^\dagger)$ are rising and lowering operators of phonons with polarization λ , $T_n^{M(\alpha)}(q, \lambda)$ are the amplitudes of transformations.

III. SPECTRA OF ELEMENTARY EXCITATIONS

As it is known, the spectra of elementary excitations of a system are determined by the poles of a Green's function, which we shall define as follows:

$$G^{\alpha\alpha'}(n, \tau; n', \tau') = -\langle \hat{T} X_n^\alpha(\tau) X_{n'}^{\alpha'}(\tau') \rangle,$$

where $X_n^\alpha(\tau) = e^{H\tau} X_n^\alpha e^{-H\tau}$, $H = H_{int} + H_{tr}$, \hat{T} is a time-ordering operator, and the averaging will be carried out with the Hamiltonian H .

As we work in a mean field approximation, for further calculations we need only "transversal" part of the Hamiltonian H_{int} , which in terms of the Hubbard's operators has the form:

$$H_{int}^\perp = -\frac{1}{2} \sum_{n,n';\alpha,\beta} \frac{I(n-n')}{2} \cdot A_i^{-\alpha} X_n^\alpha B_i^\beta X_{n'}^\beta.$$

Vector-columns A_i^α and B_i^β have the form:

$$A_1^\alpha = \begin{pmatrix} 2\gamma_{||}(-\alpha) \\ \Gamma_{||}(M) \end{pmatrix}, \quad A_2^\alpha = \begin{pmatrix} \gamma_\perp^*(\alpha) \\ 0 \end{pmatrix}, \quad A_3^\alpha = \begin{pmatrix} \gamma_\perp(-\alpha) \\ 0 \end{pmatrix},$$

$$B_1^\alpha = \begin{pmatrix} \gamma_{||}(\alpha) \\ \Gamma_{||}(M) \end{pmatrix}, \quad B_2^\alpha = \begin{pmatrix} \gamma_\perp(\alpha) \\ 0 \end{pmatrix}, \quad B_3^\alpha = \begin{pmatrix} \gamma_\perp^*(-\alpha) \\ 0 \end{pmatrix}.$$

The denominator of the Green's function, which satisfies to the Larkin's equation [8], gives the dispersion equation:

$$\det \left| \delta_{ij} + \frac{I(k)}{2} B_i^\alpha \Sigma^{\alpha\alpha'} A_j^{\alpha'} + \frac{I(k)}{2} \cdot \frac{D_\lambda(k, \omega_n) B_i^\alpha \Sigma^{\alpha\alpha'} T^{-\alpha'}(k, \lambda) T^\gamma(-k, \lambda) \Sigma^{\gamma\gamma'} A_j^{\gamma'}}{1 - D_\lambda(k, \omega_n) T^\beta(k, \lambda) \Sigma^{\beta\beta'} T^{-\beta'}(-k, \lambda)} \right| = 0. \quad (7)$$

In (7) we denote $D_\lambda(k, \omega_n) = 2\omega_\lambda(k)/[\omega^2 - \omega_\lambda^2(k)]$ is the Green's function of noninteracting phonons, $\omega_\lambda(k) = c_\lambda k$ is the law of dispersion of λ -polarized phonons and c_λ is their velocity, $\Sigma^{\alpha\alpha'}(k, \omega)$ is the nonreducible (by Larkin) part, $T^\alpha(k, \omega)$ are corresponding amplitudes of transformations. In the mean field approximation

$$\Sigma^{\alpha\alpha'}(k, \omega) = \delta_{\alpha\alpha'} b(\alpha) G_0^\alpha(\omega),$$

where $G_0^\alpha(\omega) = [\omega + (\vec{\alpha}\vec{E})]^{-1}$ is the zero Green's function, $b(\alpha) = \langle \vec{\alpha}\vec{H} \rangle_0$.

The solutions of the equation (7) determine spectra of quasiphonons and quasimagnons, however, as we study the possibility of stabilization of LMO in a 2D easy-plane ferromagnetic, i.e. the question of convergence of the integral determining fluctuations of the magnetic moment, first of all we have to obtain spectra of quasimagnons.

It is necessary to note, that the dispersion equation (7) is valid for arbitrary temperatures and values of constant of OA.

The solution of the dispersion equation in a low temperature limit allows us to determine spectra of quasimagnons:

$$\varepsilon_\beta = 2\chi, \quad \varepsilon_\alpha^2 = [E_{10} - I(k) \cdot (1 - \sin 2\delta)] \cdot [E_{10} - I(k) \cdot (1 + \sin 2\delta)], \quad (8)$$

where $E_{10} = E_1 - E_0$, ε_β is a high-frequency magnon branch, ε_α is a low-frequency magnon branch.

We study the solution (8) in the two limiting cases: the case of small and large OA value.

a). In the case of small OA value, i.e. at $\beta \ll I_0$, a low-frequency magnon branch has the spectrum:

$$\varepsilon_\alpha^2(k) = (b_0 + \alpha k^2) \cdot (b_0 + \frac{\beta}{2} + \alpha k^2), \quad (9)$$

where $b_0 = 3\lambda^2/4E$ is the parameter of ME exchange, $\alpha = I_0 R_0^2$, R_0 is the radius of interaction. In the case of the absence ME exchange we have a spectrum of an easy-plane ferromagnetic:

$$\varepsilon_\alpha(k) = \alpha k \sqrt{\frac{\beta}{2} + \alpha k^2}. \quad (10)$$

b). If OA value is large, i.e. $(\beta/4) \gg I_0$, the spectrum of quasimagnons has the form:

$$\varepsilon_\alpha^2(k) = (\frac{\beta}{2} + a_0) \cdot (\frac{\beta}{2} + a_0 - 2I_0 + 2\alpha k^2). \quad (11)$$

Here $a_0 = \lambda^2(1 + \sigma)/2E$ is the parameter of ME exchange. The spectra of quasiphonons are also determined by the equation (7), however, as they do not influence the size of the fluctuations of the magnetic moment, they are irrelevant. For example a spectrum of longitudinal quasiphonons is as follows:

$$\omega^2(k) = \omega_t^2(k) \cdot (1 - \frac{a_0}{I_0}).$$

The similar result is obtained for t -polarized quasiphonons. It is obvious, that ME interaction merely renormalizes the velocity of the sound.

IV. THE CRITICAL TEMPERATURE

Consider fluctuations of the magnetic moment, for example, $\langle (S^z)^2 \rangle$. The most simple way to calculate it is to represent the operator S^z in terms of bose operators through bosonisation of the Habbard's operators [10].

Following [10], we assign to the Habbard's operators X_n^α pseudohubbard's operators \tilde{X}_n^α , which are related with rising and lowering bose-operators of quasiparticles by the following relations:

$$\begin{aligned} \tilde{X}_n^{10} &= (1 - a_n^\dagger a_n - b_n^\dagger b_n) a_n, \quad \tilde{X}_n^{01} = a_n^\dagger, \quad \tilde{X}_n^{1-1} = (1 - a_n^\dagger a_n - b_n^\dagger b_n) b_n, \quad \tilde{X}_n^{-11} = b_n^\dagger, \\ \tilde{X}_n^{0-1} &= a_n^\dagger b_n, \quad \tilde{X}_n^{-10} = b_n^\dagger a_n, \quad \tilde{H}_n^0 = a_n^\dagger a_n, \quad \tilde{H}_n^{-1} = b_n^\dagger b_n, \quad \tilde{H}_n^1 = 1 - a_n^\dagger a_n - b_n^\dagger b_n. \end{aligned}$$

Here a is the bose operator corresponding to the transitions from a state 1 to a state 0 and vice versa, b corresponds to the transitions from a state 1 to a state -1 and vice versa.

Rewriting the Hamiltonian H in terms of bose operators, and using the fact that the one-ion Hamiltonian in the basis of its eigenfunctions becomes diagonalize:

$$H_0(n) = (\vec{E} \vec{H}_n),$$

we write down only two-partical Hamiltonian, that has the form:

$$H^{(2)} = \sum_k (E_0 - E_1 - I(k)) \cdot a_k^\dagger a_k + \sum_k (E_{-1} - E_1) \cdot b_k^\dagger b_k - \frac{1}{2} \sin 2\delta \cdot \sum_k I(k) (a_k a_{-k} + a_k^\dagger a_{-k}^\dagger).$$

Diagonalizing the obtained Hamiltonian by the standard $u - v$ transformation:

$$a_k^\dagger = u_k \alpha_k + v_k \alpha_{-k}^\dagger, \quad a_k = u_k \alpha_k^\dagger + v_k \alpha_{-k},$$

we receive:

$$H^{(2)} = \sum_k \varepsilon_\alpha \alpha_k^\dagger \alpha_k + \sum_k \varepsilon_\beta \beta_k^\dagger \beta_k,$$

where expressions for bosone energies coincide with (8).

Using the connection of spin operators with the Habbard's operators (5), for fluctuations of the magnetic moment we shall receive:

$$\langle (S_n^z)^2 \rangle = -\sin 2\delta \langle b_n^\dagger b_n \rangle + \frac{1 - \sin 2\delta}{2} \cdot \langle a_n^\dagger a_n \rangle + \frac{1 + \sin 2\delta}{2}. \quad (12)$$

Summarizing (12) we receive the desired average and

$$\frac{1}{N} \sum_n \langle b_n^\dagger b_n \rangle = \frac{1}{(2\pi)^2} \int_0^\infty \frac{k dk}{e^{\varepsilon_\beta/T} - 1}, \quad \frac{1}{N} \sum_n \langle a_n^\dagger a_n \rangle = \frac{1}{(2\pi)^2} \int_0^\infty \frac{k dk}{e^{\varepsilon_\alpha/T} - 1},$$

where $\varepsilon_\alpha(k)$ and $\varepsilon_\beta(k)$ are determined by the expressions (8). The relation (12) is valid for arbitrary values of OA constant. As in the previous case we investigate two limiting cases: small and larg OA values.

a). In the case of small OA the contribution of a high-frequency branch of excitations to fluctuations of the magnetic moment can be neglected. It is obvious, that in this case the integral

$$\int_0^\infty \frac{kdk}{e^{\varepsilon_\alpha/T} - 1}$$

converges on the lower limit ($\varepsilon_\alpha(k)$ is determined by the formula (9)), hence, in this case there exist a LMO in 2D ferromagnetics. If we disregard with the ME exchange, the integral diverges on the lower limit, that testifies to the absence of LMO. Besides from the condition $(1/N) \sum_n \langle (S_n^z)^2 \rangle = 1$ it is possible to determine the temperature of the phase transition. In our case we have:

$$\frac{1}{(2\pi)^2} \int_0^\infty \frac{kdk}{e^{\varepsilon_\alpha/T_c} - 1} = 1,$$

where T_c is the temperature of the phase transition. Substituting the expression (9) for $\varepsilon_\alpha(k)$, we obtain the expression for T_c :

$$T_c \approx 4\pi\alpha \left[\ln \frac{4\pi\alpha}{\sqrt{b_0 \cdot (b_0 + \beta/2)}} \right]^{-1}. \quad (13)$$

As it is evident from (13), the temperature of the phase transition is determined both by the ME interaction, and OA. But still ME interaction is a decisive one, and at $b_0 = 0$, $T_c \rightarrow 0$.

b). In the case of large OA value fluctuations of the magnetic moment is equal to

$$\frac{1}{N} \sum_n \langle (S_n^x)^2 \rangle = \frac{1}{(2\pi)^2} \int_0^\infty \frac{kdk}{e^{\varepsilon_\alpha/T} - 1},$$

where $\varepsilon_\alpha(k)$ is determined by the formula (11). Besides even in the absence of the ME exchange the integral does not diverge on the lower limit (in a spectrum of quasimagnons there is a finite gap at $a_0 = 0$). Evidently, such a behavior of fluctuations is connected with the fact that in the case of large OA there may realize the phase with a tensorial order parameter [11] (the so-called QU-phase).

As before, we shall determine T_c . In our case we have ($\beta/4 \gg I_0$):

$$T_c \approx \frac{\sqrt{\Delta}}{\ln(\frac{\Delta}{\pi\xi})}, \quad \xi \approx \alpha \cdot \beta, \quad \Delta \approx \frac{\beta}{2} \cdot \left(\frac{\beta}{2} - 2I_0 + a_0 \right). \quad (14)$$

V. CONCLUSION

The carried out investigations show, that the account of the ME interaction results in the stabilization of the LMO in a 2D ferromagnetic. From the formula (13) it follows, that in a low-anisotropic FM the value of T_c is determined by the ME gap of a quasimagnon spectrum.

In the case of large OA the influence of ME interaction on the establishing of LMO is not so essential, since in such systems there can exist a magnetically ordered state with a tensorial order parameter (the so-called quadrupole phase) [12]. Temperature of transition, as follows from (14) is determined mainly by the constant of OA, and the ME interaction only renormalizes it.

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